2025, Vol. 39, No. 3, 09-14

Revue Française de Gestion Industrielle
article en open accès sur www.rfgi.fr

https://doi.org/10.53102/2025.39.03.1251

ISSN: 0242-9780
eISSN: 2804-9284

How predicates can improve solving supply chain assignment problems

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Abstract: This methodological paper focuses on linear programming modelling of allocation problems, using examples of problems encountered in supply chain management. It explores the possibilities offered by software based on the AML approach (Algebraic Modelling Language), which allows the use of predicates in problem formulation, particularly to restrict the existence domain of certain variables. Binary decision variables can be used to solve assignment problems by linking objects belonging to different classes of objects corresponding to different resources or time. The constraints of the optimisation problem are then broken down into intrinsic constraints to be respected when the problem concerns only one decision (→ unique binary variable) and additional interdependent constraints involving several decisions to be made simultaneously. The intrinsic constraints, typically handled by model constraints, can be replaced by predicates associated with binary variables, allowing for both the removal of constraints and a significant reduction in the existence space of the variables, thereby greatly facilitating the solution finding of the numerical problem posed.

Keywords: Algebraic Modelling Language, Binary Variable, Predicate, Solving performance, Assignment problems, constraints typology

Comment les prédicats peuvent faciliter la résolution de problèmes d'assignation se posant dans la chaîne logistique

Résumé : Cet article méthodologique s'intéresse à la modélisation par programmation linéaire des problèmes d'assignation, à partir d'exemples de problèmes rencontrés dans la gestion de la chaîne d'approvisionnement. Il explore les possibilités offertes par les logiciels basés sur l'approche AML (Algebraic Modelling Language), qui permet l'utilisation de prédicats dans la formulation des problèmes, notamment pour restreindre le domaine d'existence de certaines variables. Les variables de décision binaires peuvent être utilisées pour résoudre des problèmes d'assignation en reliant des objets appartenant à différentes classes d'objets correspondant à différentes ressources ou à différents moments. Les contraintes du problème d'optimisation sont alors décomposées en contraintes intrinsèques à respecter lorsque le problème ne concerne qu'une seule décision (variable binaire unique) et en contraintes supplémentaires interdépendantes impliquant plusieurs décisions à prendre simultanément. Les contraintes intrinsèques, généralement gérées par des contraintes de modèle, peuvent être remplacées par des prédicats associés aux variables binaires, ce qui permet à la fois de supprimer des contraintes et de réduire considérablement l'espace d'existence des variables, facilitant ainsi grandement la recherche de solutions au problème numérique.

Mots clés : Algebraic Modelling Language (AML) ; Variables Binaires ; Prédicats ; Performance de résolution ; Problèmes d'Assignation ; Typologie des Contraintes

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Citation: Giard, V. How predicates can improve solving supply chain assignment problems. *Revue Française de Gestion Industrielle*, 39(3), 09-14. https://doi.org/10.53102/2025.39.03.1251

Historique : reçu le 12/05/2025, accepté le 25/09/2025, en ligne le 26/09/2025

1. INTRODUCTION

This paper is intended to be methodological, even though it does not contain a review of the literature (which is nonexistent for this generic modelling approach that could significantly reduce the problem size to be solved) or any models or case studies (that could utilise this approach under the conditions described below).

It suggests replacing constraints conditioning the existence of binary variables in a linear program with predicates incorporating these conditions in the definition of these variables, thereby reducing a priori their domain of existence and limiting the size of the optimisation problem to be solved, without changing the solution. The usual solution of using constraints to limit the domain of validity of a variable requires iterations in the resolution process to force the variable to comply with these constraints. The use of predicates avoids this, as the search is only performed in the relevant domain of existence of the variables, regardless of the resolution method, independent of the pre-processing implicitly performed by the predicates.

Without claiming to be exhaustive, there are three main uses of binary decision variables in supply chain management problems, which are used here to illustrate the area of application of the proposed approach, which can be used in many other contexts.

- In the first case, the binary variables are used to describe choices concerning several alternative decisions to be made by making their realisation conditional on logical conditions. A binary variable generally contains only one subscript corresponding to a simple decision number, and the model created generally includes a set of existence conditions linking these choices (incompatibility, conditional realisation, inclusion, etc.) via relations linking these binary variables. (the scientific and commercial literature lists these logical conditions (e.g., FICO, 2017)). This case is exemplified by the classic problem of selecting investments within budget constraints or scheduling problems that do not consider more than one resource shared by the activities.
- In the second case, binary decision variables are used to temporarily add additional resources to those already committed to the problem at hand.
- In the third case, binary decision variables are used to solve assignment problems that link objects belonging to different classes (in the broad sense) corresponding to resources (people, machines, sheds, factories, berths, etc.), as well as time.

This paper focuses on the third case and, incidentally, on the second to show how the use of predicates can significantly reduce the existence domains of binary decision variables. To obtain this reduction, the problem must be formulated in an optimisation software based on the principles of the AML (Algebraic Modelling Language) approach (Fourier, 2013). The AML-based software (e.g., Xpress or GAMS) combines a generic problem description, which is closely aligned with its mathematical formulation, and a dataset to create an instance of the problem to be solved. This generic description, independent of the data, can mobilise predicates in algebraic operators (Σ, Π) and universal quantifiers ($\forall$), which use parameters from the dataset in the instance creation. The created instance corresponds to the exact set of relations of the problems to be optimised, these relations integrating only variables and parameter values relevant to the specific problem to be solved.

Assignment problems link elements belonging to two sets (e.g., vessels and berth positions, or production orders and production lines) or more (e.g., vessels, berth positions, and periods). Each binary decision variable in the assignment problems corresponds to an elementary decision, assigning an entity from one set (e.g., vessel, order) to an entity from another set (e.g., berth, production line), and is constrained by two types of constraints.

- The *intrinsic constraints* on the binary variable limit the possibilities of an assignment problem that involves a single decision (e.g., a problem with a unique vessel that cannot dock at any berth at any time), which implies a problem with a single binary variable.
- In addition to the intrinsic constraints, interdependent constraints are encountered in an assignment problem involving multiple decisions (e.g., mooring a set of vessels or producing a set of

orders), which implies a problem with several binary variables. In the latter case, decisions are interdependent because they may compete for the use of limited capacity resources.

The classical formulation of the problem considers the intrinsic constraints through relations in the mathematical program. The formulation based on the AML approach avoids creating these relations by associating predicates with the binary variables in all relations that use these variables, yielding the same effect as these constraints and restricting the domain of existence of these binary variables a priori. Then, this approach limits the number of relations used by the model and decreases the resolution time, as the search is carried out on the only possible domain of existence of the binary variable, rather than on the domain defined by the Cartesian product of the cardinalities of the indices. It is worth noting that this approach often eliminates the need to develop specialised algorithms for solving problems of a specific complexity. This reduction, which may be drastic, is illustrated in Bouzekri et al. (2021, 2022, 2023).

Section 2 describes two modelling contexts that arise in the assignment problems and compares the formulations that use predicates and those that do not. In Section 3, an example from the modelling of port management (Bouzekri et al., 2023), based on real data, is used to illustrate the importance of a priori reducing the domain of existence of one of the decision variables in this model. A conclusion is proposed in Section 4..

2. FORMULATIONS WITH AND WITHOUT PREDICATES IN THE ASSIGNMENT PROBLEM FORMULATION

We will begin by presenting an example used to illustrate the passage from a formulation without a predicate to one that uses them (Section 2.1). A distinction must be made between cases where displacement is not explicitly involved in the assignment problem (Section 2.2) and those where it must be considered (Section 2.3).

2.1 Example used to illustrate the transformation of the assignment problem formulation

A binary variable has as many subscripts as there are *sets* of objects in the assignment problem, for example, "vessel $v \wedge$ berth $b \wedge$ period t", in a simplified port management problem. In this example, there are $\mathrm{V{\times}B{\times}T}$ possible binary variables x_{vpt} for a problem with V vessels, B berths and T periods. Several *attributes* can characterise each element in a set. For example, the element "vessel v" may have 3 attributes: its length l_v, its loaded draught d_v and its earliest arrival time A_v; and the element "berth b" may have two attributes: its length L_p and its depth D_p. One of these sets is privileged in the perspective retained in the assignment problem (in our example, it is the vessel) and is referred to here as *entities*.

2.2 Analysis of the displacement-ignored case in the assignment problem

After presenting the classical solution based on the introduction of explicit constraints using an example, we will examine the solution based on predicates. We then present further examples of interesting predicate use for binary decision variables, in which time is mobilised as a subscript.

2.2.1 Classical formulation using constraints

In our example, some *intrinsic constraints* on a binary variable must be considered that forbid some combinations between specific values of the attributes of each set mobilised in the multidimensional binary variable: *i*) vessel v can only dock at berth b if its loaded draught d_v is less than the depth D_b of that berth and if its length l_v does not exceed the berth length L_b; *ii*) vessel v cannot dock before its arrival A_v in port. These three intrinsic constraints are traditionally addressed by the three following relations: $\left[\sum_t \mathrm{d}_v \cdot x_{vbt} \le \mathrm{D}_b, \forall v, b\right]$; $\left[\sum_t \mathrm{l}_v \cdot x_{vbt} \le \mathrm{L}_b, \forall v, b\right]$ and $\left[\sum_b t \cdot x_{vbt} \ge \mathrm{A}_v, \forall v\right]$. In addition, all vessels must dock in the port $\left[\sum_{bt} x_{vbt} = 1, \forall v\right]$, but this temporal constraint does not link two attributes of

two sets and, therefore, cannot be viewed as intrinsic.
Introducing these constraints, whose consequence in the classical approach is to progressively restrict the search space in solving the problem, is unnecessary with predicates when using AML-based software and simplifies the problem's numerical resolution.

2.2.2 Formulation using predicates

The predicate conditioning the existence of the variable x_{vbt}, considering the same constraints, is $\left[d_v \le D_b \wedge l_v \le L_b \wedge t \ge A_v\right]$. Implicitly, this logical expression must be "True" for the x_{vbt} to be considered in the problem, discarding the other x_{vbt} for which the predicate's value is "False". With the vertical bar (|) used to denote a restriction, the constraint enforcing the vessel v to dock becomes $\left[\sum_{bt|d_v \le D_b \wedge l_v \le L_b \wedge t \ge A_v} x_{vbt} = 1, \forall v\right]$ and the 3 constraints set out in the previous section are unnecessary and can be discarded.
While using predicates, it is important to note that in every relation involving the variable x_{vbt}, the predicate conditioning its existence must be used.

2.2.3 Specific time constraints

Additional time constraints may be introduced in a problem for a subset of any resource pointed to in the binary variable. (vessels and berths, in our example). This case is illustrated below.

- Entering the port via the access channel (implicit resource) may only be possible during high tide for some heavily loaded vessels. This prohibition is controlled by the Boolean parameter O_{vt}, which is 1 at high tide for the vessels affected by the tidal restriction and 0 otherwise. For the vessels not affected by the tidal restriction, this parameter is always 1. This additional temporal constraint is considered by adding "$O_{vt} = 1$" to the previous predicate, involving the following updated relation $\left[\sum_{bt|d_v \le D_b \wedge l_v \le L_b \wedge t \ge A_v \wedge O_{vt}=1} x_{vbt} = 1, \forall v\right]$.
- This type of specific constraint introduced for a vessel can be immediately extended to any resource pointed out by the binary variable, making it possible, for example, to prohibit docking at berth b for some periods due to programmed maintenance, bad weather conditions or the temporary unavailability of certain equipment. This leads to using the Boolean parameter O'_{bt} and replacing $O_{vt} = 1$ by $O'_{bt} = 1$ in the previous predicate.
- Obviously, the predicate can take these different constraints into account simultaneously, yielding: $\left[\sum_{bt|d_v \le D_b \wedge l_v \le L_b \wedge t \ge A_v \wedge O_{vt}=1 \wedge O'_{bt}=1} x_{vbt} = 1, \forall v\right]$.

2.3 Analysis of the displacement considered the case in the assignment problem

In the example presented in the previous section, the assignment of a vessel to a berth did not consider the vessel's travel to dock at a berth.
In some assignment problems, distance (and therefore transport time or cost) must be considered. For example, the journey must be considered when a warehouse i is assigned to a factory or distribution platform j $(\rightarrow x_{ij} = 1)$, which exclusively supplies it with a specific volume of products. As displayed in the distance table H_{ij}, this transport must be considered to avoid delivering too far apart (a maximum distance of K kilometres being used). Hence, the hangar i must necessarily be assigned to a factory j via the relation (1) $\left[\sum_j x_{ij} = 1, \forall i\right]$, and the distance constraint can be considered by the relation (2) $\left[H_{ij} \cdot x_{ij} \le K, \forall i, j\right]$. This intrinsic constraint may be considered by the predicate $H_{ij} \le K$ to introduce in relation (1) $\left[\sum_{j|H_{ij} \le K} x_{ij} = 1, \forall i\right]$, making the creation of relation (2) unnecessary. In addition, interdependent constraints must be considered, such as the non-saturation of warehouse or production factory capacity; all of them must utilise this intrinsic predicate of the variable x_{ij}.
Finally, let's note that some assignment problems concern not only the assignment of an object but also its size, which leads to continuous (or discrete) variables, making the use of predicates particularly interesting. In the last example, the variable x_{ij} becomes continuous and corresponds to the volume produced by plant j for warehouse i, whose

total demand is D_j, and the relation $\left[\sum_{j|D_{ij}\le K} x_{ij}=1,\forall i\right]$ becomes $\left[\sum_{i|D_{ij}\le K} x_{ij}=D_j,\forall j\right]$ where several factories can supply a warehouse.

3. ILLUSTRATION

This example is drawn from a previous work published by Computer And Industrial Engineering, and titled *Integrated laycan and berth allocation problem with ship stability and conveyor routing constraints in bulk port* Let us focus on the first x_{vpt} of the 6 decision variables of that model, which equals 1 if vessel *v* $(v\in\mathscr{V})$ starts berthing at berthing position $b\,(b\in\mathscr{B})$ in period *t* $(t\in\mathscr{T})$, and 0 otherwise. In this model, the existence of the decision variable x_{vbt} is subject to seven conditions. For simplicity's sake, only the first four will be considered here, as the following three constraints (Channel passage at high tide of a loaded ship leaving port, berthing during working periods, and possible berthing periods becoming fewer over time) are more complicated to describe without going into the model.

- The length l_v of vessel *v* must not exceed the length L_b of berthing position *b* $\rightarrow l_v\le L_b$
- The draft d_v of vessel *v* must not exceed the water depth D_b of berthing position *p* $\rightarrow d_v\le D_b$.
- Vessel *v* must be able to berth at berthing position *b*
- Additional berthing constraint due to the temporary great difficulty of docking or loading certain boats at certain berths due to severe weather conditions (swell, storms, etc.); this prohibition is based on the Boolean parameter N_{vp}, which only worth 1 if docking is possible.
- Vessel *v* can berth only after its expected arrival time A_v, without exceeding its maximum waiting time l_v in the harbour $\rightarrow A_v\le t\le A_v+l_v$.

In a classic linear programming model, these 4 conditions must be considered by 5 constraints, which become useless if the domain of existence of the variable x_{vbt} is restricted by the predicate $N_{vp}=1\wedge l_v\le L_b\wedge d_v\le D_b\wedge A_v\le t\le A_v+l_v$.

In the case study presented in this paper, the number of periods is $T=840$ hours, the number of vessels is $N=16$, and the number of berths is $B=10$, which involves $840\times16\times10=143{,}400$ binary variables. With around 50% of the berths accessible by these vessels and all having to dock within 3 days (→ 72/840= 8.6% of the problem periods), the number of possible binary variables in this problem falls to around 6,200. Note that this number decreases further when the three ignored constraints are taken into account.

An example of using these predicates is given by the first relation in the model, which constrains the vessel *v* to dock at a single berth *p* and at a single time *t*. Of course, these predicates must be used for every instance of the variable x_{vbt} in all the model's relations:

$$\sum_{b\in\mathscr{B}|N_{vb}=1\wedge l_v\le L_b\wedge d_v\le D_b}\sum_{t\in\mathscr{T}|A_v\le t\le A_v+l_v} x_{vbt}=1,\forall v\in\mathscr{V}$$

4. CONCLUSION

Using predicates in optimisation problems with binary decision variables handled by AML-based software can significantly reduce the size of the problem posed and avoid the need to create specific solution algorithms. Furthermore, by facilitating the resolution of complex problems, researchers can take into account more realistically a certain number of constraints of the problem posed in the field (such as the simultaneous use of a fine temporal granularity to model system operation and a temporal decision whose interval can be increased for remote decisions), often ignored due to the impossibility of finding a solution numerically. It should be noted that the principles described in this paper (identification of the intrinsic constraints) can be easily adapted to other categories of optimisation problems using binary variables in any AML-based optimisation software.

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6. BIOGRAPHY

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