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Stochastic higher order macroscopic transportation modeling on road networks: managerial Implications

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Abstract: Transport systems place a key role in the development of the economic growth of countries. However, the appearance of autonomous and electric vehicles and the restrictions put in place to limit the diffusion and impacts of Covid-19ain public transport have had particularly a widespread impaction transport problems in particular at junctions. The present research helps to address these problems. This paper is concerned with stochastic traffic flow modeling on road networks, thanks to macroscopic models that belong to the so-called generic class of second order models: the GSOM family. It has been shown that such higher order models can be solved in a Lagrangian framework whose coordinates move with the traffic. The difficulty to use this resolution trick on a network is to deal with Eulerian– fixed –discontinuities such as junctions. The aim of this work is two-fold: first, to propose adapted junction models for stochastic second order macroscopic traffic flow models and second, to solve the resulting model in a moving framework. Some numerical examples are provided to show the efficiency of the approach.

Keywords: Traffic management; Stochastic traffic modelling; Lagrangian stochastic GSOM models

Modélisation stochastique macroscopique d'ordre supérieur du trafic sur les réseaux routiers : implications managériales

Résumé : Les systèmes de transport jouent un rôle primordial dans le développement de la croissance économique des pays. Cependant, l'apparition des véhicules autonomes et électriques et les restrictions mises en place pour limiter la diffusion et les impacts du Covid-19 dans les transports en commun ont eu un impact important sur l'augmentation des problèmes de transport notamment aux intersections. Le présent papier aide à résoudre ces problèmes. Cet article s'intéresse à la modélisation stochastique des flux du trafic sur les réseaux routiers, grâce à des modèles macroscopiques génériques de second ordre : la famille GSOM. Il a été montré que de tels modèles d'ordre supérieur peuvent être résolus dans un cadre lagrangien dont les coordonnées lagrangiennes se déplacent avec le trafic. La difficulté d'utiliser cette solution de résolution sur un réseau est de traiter les discontinuités eulériennes – fixes – telles que les jonctions. L'objectif de ce travail est double : d'une part, proposer des modèles d'intersection adaptés aux modèles stochastiques macroscopiques de flux de trafic de second ordre, et d'autre part, résoudre le modèle résultant dans le cadre d'un réseau routier. Quelques exemples numériques sont fournis pour montrer l'efficacité de l'approche proposée.

Keywords : Gestion de trafic ; Modélisation stochastique du trafic ; Modèles GSOM stochastique lagrangien

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1. INTRODUCTION

Transportation modeling determines the efficiency of moving products. The progress in techniques improves the reduction of congestion, delivery speed, service quality and energy saving in a context of sustainable development.

Transportation modeling takes a crucial part in the manipulation of logistic. In this paper, we are motivated by road traffic network modeling. Classically, traffic models can be divided into two categories: either microscopic or macroscopic models. Microscopic models describe the spatiotemporal behaviour of each particle or vehicle and their interactions, whereas macroscopic models describe collective properties without distinguishing individual particles.

Traffic flow models, whether macroscopic or microscopic, cannot provide a perfect representation of the physical reality. Some processes are neglected under the assumption they are negligible, others are unknowable. Even for those processes which are well understood, the collectable data may be insufficient for proper model identification. Finally, there are always stochastic perturbations affecting the flow of traffic, due to random behaviour of drivers (acceleration/deceleration, lane change...), the intrinsic stochastic character of driver interaction as various phenomena attest (spontaneous jam formation, traffic breakdown…etc) or to outside perturbations.

All these sources of errors and imperfections may be integrated into traffic models as stochastic perturbations. Driver behaviour is partly stochastic, a feature which is largely accounted for in most microscopic models.

The aim of this proposed paper is to develop a macroscopic traffic flow model for networks endowed with a tractable stochastic component. This model is compatible with both microscopic and macroscopic descriptions, and satisfies classical constraints coming from the engineering world, as for instance the invariance principle. The key idea for conciliating both microscopic and macroscopic representations is to recast the considered macroscopic model using the Lagrangian coordinates. Indeed, the Lagrangian framework focuses directly on the particles and incidentally it allows to keep track of individual behaviours (Leclercq et al., 2007).

A growing literature is devoted to stochastic modelling. Kinetic models generally contain stochastic components (lane change, desired distributions, etc) which are integrated and become deterministic elements of the model (refer to (Ngoduy, 2006) and the references therein). Similarly, some first order models such as SSMT (Lebacque, 1984) contain integrated stochastic building blocks (for instance conflicts between movements in intersections).

Other efforts have aimed at modeling directly some stochastic processes of traffic flow (Dundon & Sopasakis, 2007; Kühne & Mahnke, 2005). Some models are based on exclusion processes, such as (Dundon & Sopasakis, 2007) and (Sopasakis & Katsoulakis, 2006). The model (Tordeux et al., 2013) combines exclusion processes with a space-time discretization of the LWR model, the min formula for traffic supply and demand (Lebacque, 1996) providing jump probabilities. (Jabari & Liu, 2012) and (Jabari & Liu, 2013) follow on (Kim & Zhang, 2008) to propose a model based on the randomness of driver gap choice, and consistent with the LWR model. (Hoogendoorn et al., 2007) proposes a model of propagation of the probability of traffic breakdown along characteristic curves. (Boel & Mihaylova, 2006; Sumalee et al., 2011; Wang et al., 2007) propose stochastic perturbations of discretized macroscopic models. (Wang et al., 2007) further introduces an identification and traffic management scheme based on the stochastic approach.

Very few researches have introduced directly stochastic processes into macroscopic equations of traffic flow (Weits, 1992) for an early model). The main difficulties are computational intractability and diffusive effects. The model described in this paper introduces a model which is non-diffusive, tractable numerically, and readily interpretable from a physical point of view.

The stochastic model we propose in this work is based on the GSOM (Generic Second Order Traffic Modelling) family of models introduced in (Khelifi et al., 2017; Khelifi et al., 2015; Lebacque et al., 2007). The GSOM family of models generalizes the LWR (Lighthill-Whitham-Richards) model (Lighthill & Whitham, 1955; Richards, 1956) and encompasses many other macroscopic models. Refer to (Lebacque & Khoshyaran, **A**2013) for examples. GSOM models combine a conservation equation for the density (accounting for kinematic waves in traffic) with a system of conservation laws for the behavioural attributes of individual cars and drivers, such as vehicle type, aggressiveness, destination, information flow to and from a vehicle, etc.

The main idea of the paper is that random perturbation affects vehicle dynamics indirectly, by affecting the driver behaviour. Thus, random perturbations will be described by a specific driver attribute. Since this attribute is Lagrangian, its dynamics can be described by a stochastic differential equation in Lagrangian coordinates. This property, combined with the properties of GSOM models, ensures the tractability of the model, although it is expressed as a system of stochastic partial differential equations in Eulerian coordinates.

The article is organized as follows. Section 2 explore the theoretical and practical traffic management implications. In Section 3, the generic class of second order macroscopic traffic flow models called GSOM family is introduced. In section 4, we discuss the stochastic GSOM model. In Section 5, we briefly review and discuss the literature for solving GSOM models posed on junctions. Our aim is to show that the Lagrangian framework is well-suited for designing the solution to GSOM problems even if incorporating discontinuities. The complete numerical methodology on junction is described in Section 5 and some numerical examples are given. Finally, we provide some conclusions on this work and give some insights on future research in Section 6.

2. GSOM FAMILY

2.1 Specification of GSOM models

First order traffic flow models, such as the seminal LWR model standing for (Lighthill, & Whitham, 1955; Richards, 1956) have been used for quite a long time for modelling traffic flows on networks (Garavello & Piccoli, 2006; Lebacque & Khoshyaran, 2005) for respectively mathematical and engineering reviews). However, first order models generally fail to recapture accurately specific and meaningful traffic flow phenomena such as stop-and-go waves, the capacity drop or the bounded acceleration. Indeed, first order models assume that every driver behaves in the same fashion that does not allow to reproduce phenomenon created by the variability of the drivers. Thus, we focus here on the Generic Second Order Models (GSOM) family (Lebacque et al., 2005; Lebacque et al., 2007).

The main GSOM variables are: time t, position x, density ρ, flow q, speed v, and attribute I (it can be related to the type of vehicle, driver behaviour, origin/destination…).
The GSOM model combines the conservation equation with the fundamental diagram of specific driver behaviour attribute.
The model can be stated in conservation form as follows:

$$\begin{cases} \partial_t \rho + \partial_x (\rho v) = 0 & \text{Conservation of vehicules} \\ \partial_t (\rho I) + \partial_x (\rho I v) = 0 & \text{Conservation of } I \\ v = \Im(\rho, I) & \text{Fundamental diagram (FD), dependent on } I \end{cases} \quad (1)$$

x

t

I is a Lagrangian driver attribute that characterizes the behaviour of each driver. It is preserved along the trajectories of vehicles, a result in harmony with the fact that contact discontinuities waves propagate discontinuities of I to the speed.

The GSOM family encompasses a large variety of higher order traffic flow models such as the LWR model itself which is simply a GSOM model with no specific driver attribute, the LWR model with bounded acceleration (Lebacque & Khoshyaran, 2002; Lebacque, 2003; Leclercq et al., 2007), the

ARZ model (Aw and Rascle, 2000; Zhang, 2002), the Generalized ARZ model proposed in (Fan et al., 2013) as a special case of (Zhang et al., 2009), the multi-commodity models (multi-class, multi-lanes) (Bagnerini & Rascle, 2003; Herty et al., 2008; Jin & Zhang, 2004; Klar et al., 2003), the Colombo 1-phase model (Lebacque et al., 2005; Lebacque et al., 2007) or the stochastic GSOM model (Khoshyaran & Lebacque, 2009).

GSOM models have been already well studied on homogeneous sections but they have attracted little attention for their implementation on junctions. However, junctions are the main source of congestion for traffic streams on a network.

2.2 Lagrangian expression of the GSOM family

The common expression of GSOM models in Eulerian coordinates (t,x) is given by equation (1). However, it is well-known that Lagrangian framework (t,n) is particularly convenient for dealing with flows of particles and it is especially true in traffic flow modeling (see (Leclercq et al., 2007; Van Wageningen-Kessels et al., 2013) and references therein).

The first fundamental variable of the Lagrangian GSOM model is the number of cumulated vehicles N. This variable is supplemented by a second variable which is the time T.

Considering that $x_n(t)$ the trajectory of the nth vehicle, the Lagrangian form of the GSOM model is given by:

$$\begin{cases} x_n(t+\Delta t) = x_n(t) + \Delta t . v_n(t) \\ v_n(t) = \Im\left(\dfrac{x_{n-1}(t) - x_n(t)}{\Delta N}, I_n(t)\right) \\ I_n(t+\Delta t) = I_n(t) + \Delta t . \varphi\big(I_n(t)\big) \end{cases} \tag{2}$$

Where $\varphi\big(I_n(t)\big) = -\dot{I}_n(t)$ and $I_n(t)$ represents the invariant associated to the nth vehicle.

In the second case, the appropriate numerical scheme is defined as follows:

$$\begin{cases} r_n(t+\Delta t) = r_n(t) + \dfrac{\Delta t}{\Delta N} . \big(v_{n-1}(t) - v_n(t)\big) \\ v_n(t) = \Im\big(r_n(t), I_n(t)\big) \\ I_n(t+\Delta t) = I_n(t) + \Delta t . \varphi\big(I_n(t)\big) \end{cases} \tag{3}$$

Where r is the discrete particle spacing. Then the scheme shown in Equation (3) is deduced from Equation (2). Note that both schemes are first order schemes. The first discrete model (Equation (2)) is an explicit Euler scheme and the second scheme (Equation (3)) can be interpreted as the seminal Godunov scheme (see (Godunov, 1959)) applied with demand and supply.

3. THE STOCHASTIC GSOM MODEL

3.1 Specification of stochastic GSOM model

The fundamental idea of the model is to consider the driver attribute I as a variable which is stochastic, as the result of random interactions of the driver with other drivers.

The dynamics of the attribute I are described as follows:

$$\dot{I} = \varphi\left(I, \frac{d\mathrm{B}_t}{dt}\right) \tag{4}$$

B_t is a Brownian process (and $\dfrac{d\mathrm{B}_t}{dt} \overset{def}{=} W_t$ the corresponding white noise process).

The resulting stochastic model in natural (x,t) coordinates can be stated as:

$$\begin{cases} \partial_t \rho + \partial_x(\rho v) = 0 & \text{Conservation of vehicules} \\ \partial_t(\rho I) + \partial_x(\rho I v) = \rho \varphi(I, W_t) & \text{Conservation of } I \\ v = \Im(\rho, I) & \text{FD dependent on } I \end{cases} \tag{5}$$

The driver attribute includes two components, each of which can be a vector:

- a stochastic component, which will express the impact of random perturbations, the impact of the imperfections of the modelling process, and the error process affecting various parameters of the model (case of a model identification or traffic control problem).

- a deterministic component, which will have the usual interpretation of GSOM driver attributes (the propensity to more or less aggressive behaviour, the driver destination, the driver class, the parameters of the fundamental diagram, or a parameter controlling the relaxation of traffic towards some reference dynamics).

3.2 Lagrangian expression of the stochastic GSOM model

Let us introduce Lagrangian coordinates, N the vehicle index and T the time, and let $r \overset{def}{=} 1/\rho$ denote the spacing. I is a random variable depending on the vehicle index N (it is a driver attribute) and on the random event W_t.

In Lagrangian coordinates (N,T), the stochastic GSOM model (5) can be expressed as:

$$\begin{cases} \partial_T r + \partial_N v = 0 & \text{Conservation of vehicules} \\ \partial_T I = \varphi(I, W_T, N) & \text{Dynamics of } I \\ v = V(r, I, N) & \text{Driver dependent FD} \end{cases} \tag{6}$$

r

$\varphi = 0$

$\varphi(I,r) \neq 0$

Note that $V(r,I) \overset{def}{=} \Im(1/\rho, I)$.

The process I is described by the stochastic ordinary differential equation.

$$\partial_T I = \varphi(I, W_T, N) \tag{7}$$

and is assumed to be driven by the brownian motion B_t.

The complete Godunov discretization is summarized in (Lebacque, J.P., and Khoshyaran, M.M., 2013) by:

$$\begin{vmatrix} x_n(t+1) = x_n(t) + \Delta t v_n(t) \\ v_n(t) = V(r_n(t), I_n(t)) \\ I_n(t+1) = L(I_n(t), t, t+\Delta t; W_t) \end{vmatrix} \tag{8}$$

3.3 Example of stochastic GSOM model

The simplest model for (4) is the linear model:

$$\varphi(I, W_t) = -\alpha I + \sigma W_t \tag{9}$$

With α and σ two parameters and $\frac{d\mathrm{B}_t}{dt} \overset{def}{=} W_t$ the white noise process. The idea of expression (9) is: the dynamics of I_n results from two competing processes: a relaxation process and a white noise perturbation process.

The expression of the semi-group L is easily deduced:

$$L\left(I(t), 0, t; W_t\right) = \sigma \int_0^t e^{-\alpha(t-s)} dB_s\left(W_t\right) + I(0) e^{-\alpha t} \tag{10}$$

The expression (10) defines the solution of (9) as an Ornstein-Uhlenbeck process for I (see the Godunov particle discretization in Lebacque & Khoshyaran, 2013).

The CFL condition can be obtained by expressing that if the distance between particle n and $n-1$ is greater than the minimum headway $r_{\min}(I_n(t))$, and it stays so after one iteration:

$$r_{\min}(I_n(t)) + \Delta t V(r_n(t), I_n(t)) \leq x_{n-1}(t) - x_n(t)$$

The following constraints results, which is applied in the simulation runs in this paper:

$$\Delta t \leq \mathrm{Max}_{\left[\substack{r \geq r_{\min}(I) \\ -I_* \leq I \leq I^*}\right]} \left(\frac{1 - r_{\min}(I)/r}{R(1/r, I)} \right) \tag{11}$$

4. NETWORK MODELING BASED ON STOCHASTIC GSOM MODEL

4.1 Review of the literature

In this paper, we are motivated by road network modeling, thanks to stochastic GSOM models. First order traffic flow models have been used for quite a long time for modelling traffic flows on networks (see Garavello & Piccoli, 2006; Lebacque & Khoshyaran, 2002 for instance). In particular, the seminal LWR model standing for (Lighthill & Whitham, 1955; Richards, 1956) has been widely used. However, first order models do not allow to recapture accurately specific and meaningful traffic flow phenomena. Thus, we focus on the stochastic GSOM models family which encompasses a large variety of higher order traffic flow models.

Stochastic GSOM models have been already well studied on homogeneous sections but they have attracted little attention for their implementation on junctions, as it is discussed in this section.

However, junctions are the main source of congestion for traffic streams on a network.

There already exist a few works on the Lagrangian modeling of junctions based on GSOM models; see for instance (Khoshyaran & Lebacque, 2008; Van Wageningen-Kessels et al., 2013). However, some of these works are based on very specific examples extracted from the GSOM family. But it is not straightforward to extend the numerical methodologies presented in these papers to the stochastic GSOM model. In (Khoshyaran & Lebacque, 2008), the authors consider the Godunov scheme and extend this particle discretization to networks, addressing the problem of junction modeling through a supply-demand approach. The authors make the choice to introduce an internal state model (Khoshyaran & Lebacque, 2009; Lebacque et al., 2008) and assume that the particles share the same attribute once they have passed. The authors deal also with densities and flows which is not particularly convenient with GSOM models in the Lagrangian framework. Hopefully, dealing with spacing instead of density will ease the resolution of the model. While boundary conditions can be treated within the framework of supply-demand flows methodology (Khoshyaran & Lebacque, 2008; Lebacque et al., 2007), expressions of upstream and downstream boundary conditions into Lagrangian coordinates can be obtained in the framework of variational approach for GSOM models (Lebacque & Khoshyaran, 2013). It will be developed in this section.

In order to model traffic on a network it is necessary to interface traffic flow on links and nodes. Our approach is to use an internal state node model (Lebacque & Khoshyaran, 2005; Lebacque, 2003). The node is viewed as a point (its physical dimension is neglected) but traffic inside the node is disaggregated per movement. Simple conservation and FIFO propagation processes describe the traffic dynamics per movements.

Each node is endowed with behavioural properties which are expressed by node supplies and demands (Lebacque & Khoshyaran, 2013; Lebacque, 1996; Lebacque, 2003). These will be adapted to the stochastic context. The flow of traffic inside the node is discretized into particles as it is on links, which yields exact attribute dynamics. The driver attributes are averaged per movement for the estimation of these node supplies and demands. The node in- and out-flows are calculated according to the min formula for supplies and demand (Lebacque & Khoshyaran, 2005) and therefore the model satisfies the invariance principle. The in- and outflows are converted into particle dynamics, expanding on (Lebacque & Khoshyaran, 2013). Each node acts like a buffer between its upstream and downstream links which will be the subject of this section.

4.2 Methodology for the Lagrangian modeling of nodes

In Lagrangian, one may expect that the upstream demand is given by the speed of the next vehicle which will pass through the junction. The difficult point is that the speed is computed with respect to the spacing with the leader vehicle. If the leader vehicle gets into the junction, the following vehicle has no more leader vehicle. The idea is then to assume a point-wise junction model with an internal state (first introduced in Khoshyaran & Lebacque, 2009) that is used as a buffer between incoming and outgoing branches of the junction. We recall that this buffer has internal dynamics and we can define an internal supply which depends on the number of stored vehicles. To solve the problem of defining a spacing when the leader vehicle has entered the junction point, we set a new speed function which only depends on the distance to the junction (which is assumed to contain the “fictitious” leader vehicle). We also need to particularly pay attention to the specific case of $r \leq r_{crit}$ where r_{crit} is the critical spacing under which speed is almost null. Indeed, in this case we would like to avoid to block vehicles which does not make sense if r denotes the distance to the junction here.

In this section, we describe the numerical scheme adapted for the generic stochastic GSOM model (equation (8)) posed on junction. We set a junction as the union of incoming and outgoing links that intersect at a unique point called the junction point.

In the beginning, it is necessary to converse the supply and demand functions classically expressed in Eulerian coordinates denoted respectively by Δ and Σ into Lagrangian ones. In Eulerian, the passing flow q is given by the minimum between upstream demand and downstream supply.

For deducing a Lagrangian discretization of a traffic flow model on a junction, it is necessary to take into consideration different elements:

- The link model, which is given by Equations ((2) or (3)) and (8);
- The upstream (resp. downstream) boundary conditions for any incoming (resp. outgoing) link;
- The internal junction model, say the way particles are assigned from incoming road to outgoing road and eventually the internal dynamics of the junction point;
- Link-junction and junction-link interfaces.

These constituting elements are addressed in what follows.

4.2.1 Downstream boundary condition

Consider the downstream boundary of a given outgoing road j located at x_S. The downstream boundary data at x_S is given by the downstream supply σ^t at time step t. (n) is the last particle located on the link (or at least a fraction $\eta\Delta N$ of it is still on the link, with $0<\eta\leq 1$). See (Figure 1).

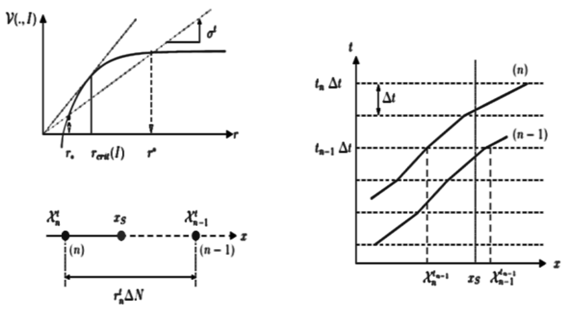

Figure 1: Downstream boundary conditions

Let us define by t_{n-1} the first time step following the exit of particle (n − 1) (for any n ≥ 2). It denotes the exit time of the end of the particle $n-1$, say, the minimal time t_{n-1} for which:

$$\chi_{n-1}^t \geq x_s \text{ for any } t \geq t_{n-1}$$

It is noteworthy that the CFL condition (15) prevents that two vehicles (or more) exit the link in the same time step.

Then, the algorithm is composed as follows:

1. First, compute the spacing r_n^t associated to the particle (n) such as:

$$r_n^t = \frac{\chi_{n-1}^t - \chi_n^t}{\Delta N}$$

where $t = t_{n-1}$ and $\chi_{n-1}^t \geq x_s$ is the last position computed for the leader particle (n − 1).

2. Then, initialize μ as follows:

$$\mu = \frac{x_S - \chi_n^{t_n}}{r_n^{t_n}\Delta N}$$

While the particle (n) has not totally exited the link, i.e. while $t \leq t_n$, we redefine the spacing as follows:

$$r_n^t = \frac{x_S - \chi_n^t}{\mu\Delta N}$$

3. We have to distinguish two cases:

- Either $V\left(r_n^t, I_n^t\right) \leq \sigma^t r_n^t$: in this case, the downstream supply is sufficient to accommodate the demand on the link and the spacing r_n^t computed at the previous step is conserved.
- Or $V\left(r_n^t, I_n^t\right) > \sigma^t r_n^t$ (unmet demand): in this case, the demand on the link cannot be fully satisfied since the downstream supply limits the outflow. Then, we have to modify r_n^t by selecting the smallest root r_* (see (Figure 1)) of:

$$V\left(r, I_n^t\right) = \sigma^t r$$

It means that we select the solution corresponding to the congested phase (see Figure 1) and we update $r_n^t \leftarrow r_*$.

4. Finally, we update the position and the attribute of particle (n) using equation (8)

$$\begin{cases} x_n(t+1) = x_n(t) + \Delta t v_n(t) \\ I_n(t+1) = L(I_n(t), t, t+\Delta t; W_t) \end{cases}$$

5. One can face the two following cases:

- if the particle (n) has not totally exited the link, say, if $\chi_n^{t+1} < x_s$, then we need to update the fraction μ as follows:

$$\mu \leftarrow \mu - \frac{\Delta t}{r_n^t \Delta N} V\left(r_n^t, I_n^t\right)$$

and iterate on the time step $t \leftarrow t+1$ and redo steps 2 to 4.

- conversely, if the particle (n) has totally exited the link, say, if $\chi_n^{t+1} \geq x_s$, then we need to update both the time step $t \leftarrow t+1$ and the particle index $n \leftarrow n+1$ and then restart at step 1.

This ends the sub-routine for the treatment of a downstream boundary condition.

4.2.2 Upstream boundary condition

Consider the upstream boundary of a given incoming road i located at position x_E. The upstream boundary data at x_E is given by the discrete prescribed upstream demand δ^t at time step t. Let (n) be the last particle that has totally entered the link at time t_n, with $n \geq 1$. Assume that $t_n = (t-1+\varepsilon_n)\Delta t$ for a $0 < \varepsilon_n \leq 1$. Set t as the current time step and assume that $t\Delta t \geq t_n$. Notice that we necessarily have $t > 1$ since the particle (n), with $n \geq 1$, has already entered the link.

Unlike for the downstream boundary condition where we know exactly the position of the last particle which has exited the link (Section 5.2.1), we do not know precisely the position of the next particle, labelled (n + 1), which will enter the link.

Thus, we have to correctly compute the entry time of each particle (Figure 2). These entry times can be computed thanks to the effective cumulative flow at the upstream entry, denoted by q^t for any time in $\left[t\Delta t, (t+1)\Delta t\right[$. If one considers a fictitious junction model just upstream the entry point, in which particles are stored before being injected into the link whenever it is possible, then we can deduce a stock model which is similar to an internal junction model.

Initially, the stock is equal to zero.

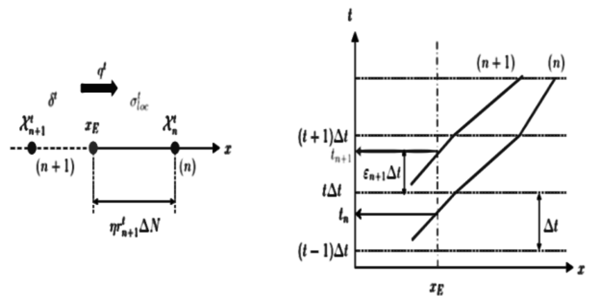

Figure 2: Upstream boundary conditions

Assume that the next particle (n + 1) will enter in the link at time $t_{n+1} = (t+\varepsilon_{n+1})\Delta t$ for a $0 < \varepsilon_{n+1} \leq 1$. Let us introduce $\eta(0 < \eta \leq 1)$ the fraction of the particle (n + 1) which has already got into the ink at time step t satisfying $t\Delta t \geq t_n$.

The algorithm is composed as follows:

1. Instantiation:

- Assume that the entry time t_n of particle (n) and the flow q^{t-1} on $\left[(t-1)\Delta t, t\Delta t\right[$ are known. We initialize the fraction μ as follows:

$$\eta = q^{t-1} \frac{(t\Delta t - t_n)}{\Delta N}$$

While the spacing r_{n+1}^t is given by:

$$r_{n+1}^t = \frac{\chi_n^t - x_E}{\eta \Delta N}$$

- We introduce the local supply immediately downstream the entry point x_E as:

$$\sigma_{loc}^t = \Xi\left(\frac{1}{r_{n+1}^t}, I_{n+1}^t, I_n^t; x_E\right)$$

- We denote by F^t the number of particles stored inside the fictitious junction.

2. Stock model: The evolution of the stock F^t is given by:

$$F^{t+1} = F^t + \left(\delta^t - q^t\right)\Delta t$$

Where δ^t is the cumulative demand, q^t is the effective flow of particles which enters the link. With a simple test, we can distinguish two cases:

- if $F^t > 0$, then there is a (vertical) queue just upstream the entry point and we get:

  $$q^t = \min\left\{\sigma_{loc}^t, Q_{\max}\left(I_{n+1}^t\right), \frac{F^t}{\Delta t} + \delta^t\right\}$$

  where $Q_{\max}$ is the maximal flow obtained for the flow-density fundamental diagram corresponding to the attribute I_n^t.

- if $F^t = 0$, then there is no queue and the flow is simply given by the minimum between the local upstream demand δ^t which is given and the local downstream demand σ_t, say:

  $$q^t = \min\left(\sigma_{loc}^t, \delta^t\right)$$

3. Computation for the following time step and generation of the next particle:

The particle (n + 1) is totally generated at time $\left(t+1\right)\Delta t$ if and only if we comply to:

$$\mu\Delta N + q^t\Delta t \geq \Delta N$$

At time step t, there are $\left(1-\mu\right)\Delta N$ vehicles from the particle (n + 1) that remain in the fictitious buffer. So, we distinguish two cases:

- If $q^t\Delta t < \left(1-\mu\right)\Delta N$, then the particle $\left(n+1\right)$ does not completely extend the node and we update μ as follows:

  $$\mu \leftarrow \mu + \frac{q^t\Delta t}{\left(1-\mu\right)\Delta N}$$

  The considered particle index stays unchanged.

- If $q^t\Delta t \geq \left(1-\mu\right)\Delta N$, then $\left(n+1\right)$ has completely entered the link and the entry time is $t_{n+1} = \left(t + \varepsilon_{n+1}\right)\Delta t$ where:

  $$\varepsilon_{n+1} = \frac{\left(1-\mu\right)\Delta N}{q^t\Delta t}$$

The position of particle (n + 1) is thus updated as follows:

$$\chi_{n+1}^{t+1} = x_E + \left(1-\varepsilon_{n+1}\right)\Delta t V\left(r_{n+1}^t, I_{n+1}^t\right)$$

We also update the particle index $n \leftarrow n+1$.

4. Final update: We compute the attribute at time step (t + 1):

$$I_{n+1}^{t+1} = L\left(I_{n+1}^t, t, t+\Delta t; W_t\right)$$

and finally, we itemize by updating the time step $t \leftarrow t+1$ and we restart the sub-routine from Step 1.

We note that this methodology can be directly applied to treat any junction-link interface as we will see in next Section.

4.2.3 Internal state junction model

We consider a point-wise junction model with an internal state (Khoshyaran & Lebacque, 2009) that is used as a buffer between incoming and outgoing branches of the junction. We recall that this buffer has internal dynamics and we can define an internal supply which depends on the number of stored particles. It assumes that the junction $\left(z\right)$ has a physical dimension and acts as a buffer and those vehicles are stored before extending the outgoing branches. The internal state has specific attributes such as: $N_z\left(t\right)$ is the total number of vehicles in the node, $N_{z,j}\left(t\right)$ is the number of vehicles registered in the node and destined to link $\left(j\right)$ and $I_z\left(t\right)$ is the driver attribute of vehicles stored in the node.

Notice that the link-junction (resp. junction-link) interface is treated as a downstream (resp. upstream) boundary condition. Thus, we apply the algorithms described above, considering the local supply (resp. demand) of the buffers inside the junction point which are defined according to the number of stored particles.

There exists different strategies to deal with the assignment of particles through the junction. They are detailed here below.

- Assume that we have only the information of assignment of particles say the matrix $(\gamma_{ij})_{i,j}$ that describe the proportion of particles coming from any road $i \in \mathrm{I}$ that want to exit the junction on road $j \in J$. In that case, we consider that a particle that enters the junction from road i will exit on road j with a probability of $\alpha_{i,j}$.
- Assume that path through the junction, say the number of the outgoing 1 branch on which the particle n will exit, is directly included in the particle attribute $I(t,n)$ and that this information does not evolve in time. In that case, the choice of the outgoing branch for particle (n) is straightforward.
- Assume now that we consider a global network with many arcs and many junctions. We can imagine that the particle attribute $I(t,n)$ encompass the origin-destination information for particle n . This information can depend on time, for example if the particle changes his mind about the path according to the traffic states on the network. One can assume that we can build a reactive assignment model that give us the path followed by particles. This model can be coupled with another model of command, supposed to be governed by a traffic planning agency for instance. Let imagine that the central planners collect information on travel times on each arc of the network and that these travel times are displayed for particles that enter the network. Then any particle will select the appropriate path at each junction for going to their destination.

Moreover, we can distinguish two different cases for describing the internal dynamics of the junction. Indeed, one can consider that once particles have entered the junction, whatever are their origins, they are immediately assigned to the buffer corresponding to their wished outgoing branch $j \in J$. But it is also possible to consider that inside the junction point, any particle has a non-trivial travel time before to join their exit, which can be affected by the total number of particle inside the junction point or by the "physical" conflicts that can appear between the internal lanes of the junction point.

4.3 Numerical example

We consider a network made of 4 junctions with 4 incoming and 4 outgoing roads. Each junction has two incoming and two outgoing links (Figure 3). The considered simulation site corresponds to a road network with two lanes. We consider for this numerical example the Ornstein-Uhlenbeck stochastic GSOM model described in section (4.3).

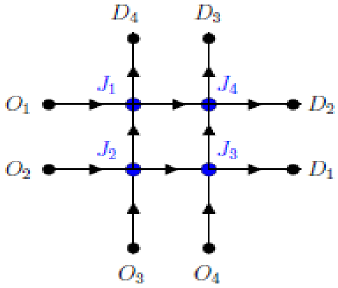

Figure 3: Example of the network study

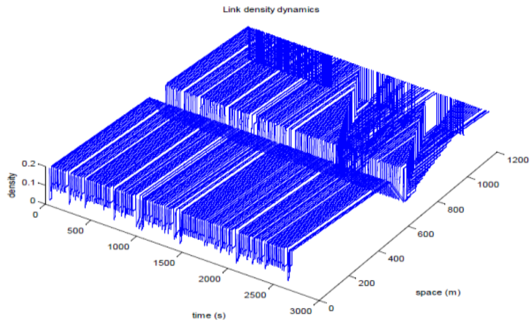

Figure 4: Link density dynamics

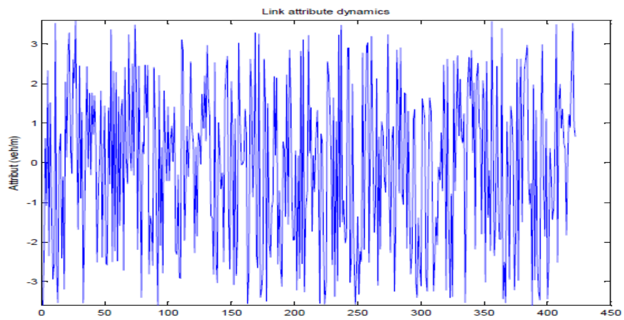

Figure 5: Link attribute dynamics

The example in (figures 4-5) shows traffic dynamics on the network shown in figure 3, with the interplay of kinematic waves (density) and contact discontinuities (driver attribute waves). (Figures 4 and 5) show the dynamics of the total number of vehicles in the network, as well as the mean driver attribute and mean critical number of vehicles (dependent on the driver attribute) inside nodes. The reader can notice that our numerical method can accurately recapture the shock wave due to the congestion and then the rarefaction wave, due to the decrease of the upstream demand, that mitigates the traffic jam later on.

5. TRAFFIC MANAGERIAL IMPLICATIONS

Transport systems place a key role in the development of the economic growth of countries. However, it was only from the middle of the previous century that they have become a serious problem. The appearance of autonomous and electric vehicles and the restrictions put in place to limit the diffusion and impacts of Covid-19 in public transport have had particularly a widespread impact on people’s lives, and the way energy is used across entire economies. Therefore, there is a change in the transportation system. The crisis has affected public transport (buses, trains and planes) nationally and internationally. It has witnessed a colossal increase of private vehicles on the roads. Unfortunately, the infrastructure of roads and traffic systems has not kept pace with this growth, resulting in inefficient traffic management. Owing to this imbalance, traffic jams on roads, congestions, and pollution have shown a marked increase. As a result, transport problems are becoming increasingly complex. This complexity was primarily due to the growing motorization and its consequences on the dramatically increasing traffic congestion. Indeed, traffic congestion is a critical societal problem due to the social - economic and environmental problems that it generates. Traffic congestion in urban road and freeway networks leads to a strong degradation of the network infrastructure and accordingly reduced throughput, which can be countered via suitable control measures and strategies.

Thus, it appears urgent and necessary, particularly with the emergence of the concept of sustainable development, to develop short- and medium-term solutions to reduce these traffic congestion effects; which requires a growing sophistication of traffic management (for instance through the construction of new infrastructure or the public transport development, or the straightforward and most effective solution which is the infrastructure optimization (Lesuseur-Cazé et al., 2022)). Traffic modelling plays a crucial role in traffic management (Bara, 2021). It can be applied to plan and manage the traffic within certain road network (May, 1990). For example, making a smooth traffic at an intersection (Lebacque & Khoshyaran, 2005). Road traffic control is the only effective solution to optimize the efficient use of transport infrastructures in order to reduce costs and accident risks, and negative effects of pollution.

The management of growing traffic is a major issue across the world. Road traffic control systems (freeway networks, and route guidance Intelligent Transportation Systems, Intelligent Traffic Signals Installation, selected application results, obtained from either simulation) have a great potential in offering solutions to such issues by using new technologies and traffic flow models.

The development of traffic flow models; able to describe, explain and predict vehicle interactions is a precondition for the development and evaluation of traffic control systems and road traffic management solutions. Therefore, traffic flow modeling and numerical simulation have an increasingly important role in the traffic flow optimization by reducing traffic congestion. The traffic flow models-based solutions for traffic management and control have been categorized as traffic data collection solutions, traffic management solutions, congestion avoidance solutions, key strategies based on machine learning and computational intelligence for avoiding congestion, important solutions for accurately predicting travel time, and travel time prediction solutions.

Principally, simulation and traffic flow models focus on three output values to solve traffic problems (Friedrich, 2015).

Firstly, in the traffic flows alternative routes can be identified based on the number of vehicles. By using the simulation traffic flow model, transport modeller can devise on how to reduce the levels of congestion of certain roads.

Secondly, network element in traffic simulation consists of links and intersections (Barcello, 2010). This is related to the geometric layout of the road. Using appropriate simulation software and traffic flow model by infrastructure planner, the road geometric design can be changed to see how it can influence the current traffic situation.

Thirdly, dynamic models can help to estimate the time and cost of travel (for network managers to adjust red lights, for example). This is especially used when the assessment of transport improvement is needed to be measured. The regional traffic planner can easily make a performance comparison without any extra cost of money and time (for example for travel time estimation (Princeton & Cohen, 2011), for speed limit changes on urban motorways using a first order macroscopic traffic simulation tool (Cohen et al., 2014), or for intelligent transport system Gertrude Saem (a regulation system of multimodal urban traffic in real time, using a macroscopic traffic model : Gertrude SAEM, 2018)).

Therefore, the objective of simulation model is to presents a real traffic situation in to dynamic model. It is in this perspective that traffic flow theory resulted in the development of a spectrum of traffic flow models (sub-microscopic, microscopic, macroscopic and mesoscopic models). The main objective of these traffic flow models is to allow the traffic control strategies development (traffic light management systems (Khelifi et al., 2015), intelligent traffic management strategies, efficient management of traffic flows at intersections, traveler information systems, real-time monitoring and control systems, connected vehicles, and automation).

In addition, intersection modeling and boundary conditions analysis are extremely important for flow traffic models because they are the keys to: the improvement of the identification and the calibration of traffic simulation models, the large and complex networks modeling, control traffic management applications, the understanding of drop in capacity.

That's why we chose to clarify in this study one of the main scientific issues, which is the intersections traffic modeling. In this paper, we developed a junction model which is compatible with microscopic and macroscopic descriptions to optimize the traffic management solutions. The microscopic representation of traffic flow is particularly suited for traffic management methods, while staying compatible with a macroscopic representation allowing global evaluation.

The key idea for conciliating both microscopic and macroscopic representations is to recast the macroscopic model under its Lagrangian coordinates. Indeed, the Lagrangian framework focuses directly on the particles and incidentally it allows to keep track of individual behaviors.

6. CONCLUSION

In this paper, we have discussed a totally new numerical method to deal with the generic class of stochastic second order macroscopic traffic flow models, known as the GSOM family, posed on a junction. The generic GSOM model is recast in the Lagrangian framework and we have a careful look at the boundaries conditions for links and junctions. The Godunov particle discretization (14) constitutes a convergent discretization for almost all $W_t \in \Omega$, and provides an intuitive description of the model solutions. The model described in this paper reproduces the variability of trajectories and predicts breakdown of traffic (Figure 4-5).

The aim of this paper is to extend the stochastic GSOM model to intersection modeling, in order to check whether the model (5) predicts breakdown of traffic and similar phenomena.

Moreover, this research has great potential in terms of improving traffic and presents real managerial insights with important implications. This research is important specifically in the context of new logistics challenges such as: the growing importance of using alternative modes of transportation (for examples: electric and autonomous vehicles); Covid impact on public transportation; …etc.

By the way, we highlight below some interesting research directions. As a future research perspective, we can apply our model to a real dataset and value it in terms of costs or traffic performance improvement.

Ongoing research addresses to deduce the resulting stochastic properties of flow and density, in order to apply the model to stochastic traffic control. The discrete model (6) can be replaced by more complex time integration schemes (Runge-Kutta schemes, the trapezoidal scheme,…etc). Such numerical schemes can be justified mainly if we consider a source term at the r.h.s. in (10) which is not null, say $\varphi(I)\neq 0$ or depends on r, say $\varphi(I,r)$. In the particular case of $\varphi=0$, explicit Euler scheme is very satisfying. Moreover, it is also imaginable to build an implicit scheme, even if it means a higher computational cost.

Further perspectives which will be developed in future works include the study of error processes for model parameters and measurements as attributes and their use for data assimilation and model identification on networks.

One another direction of research would be to compare the numerical results obtained with our monotone scheme and those obtained from the variational approach for GSOM models (and a fortiori for the LWR model : Lebacque & Khoshyaran, 2013) is rather complex to apply on real network with many links and when applied to a model with for example a non-triangular fundamental diagram, it looses the advantages of higher accuracy. For the future perspectives of this research, we can also explore the impact of the proposed modelling on a real case. For example, this impact can be expressed in terms of saved CO2 emissions, minimised costs, improved traffic flexibility (Benzidia, 2012; Benzidia, 2014)... etc.

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